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B.Sc.-part-I (H), Paper-I
Set Theory Date- 21-04-2020.

Q.1 Define partial order, linear order and well order on a set with an example of each one of them.

Ans. Partial Order: → Let X is any non-empty set and R is a relation in X then R is called a partial order relation in X when R is antisymmetric and transitive. If we write R as $\leq$, then '$\leq$' is a partial order if:

(I) For every $x, y \in X$, if $x \leq y$ and $y \leq x$ then $x = y$ [anti-symmetry]

(II) For every, $x, y, z \in X$, if $x \leq y$ and $y \leq z$ then $x \leq z$ [transitivity]

The set $(X, \leq)$ is then called a partially ordered set, we shall take $x \leq y$ and $y \geq x$ as synonymous. For simplicity $(X, \leq)$ is written as X.
For any set X, the inclusion relation $\subseteq$ is a partial order in $P(X)$.

Linear order: → If for every $x, y \in X$, $x \leq y$ or $y \leq x$ then $\leq$ is said to be ~~total~~ linear order relation in X.
For example: In the set $X = \{1, 2, 3\}$, $A = \{1, 2\} \in P(X)$ and $B = \{2, 3\} \in P(X)$, but neither $A \subseteq B$ nor $B \subseteq A$.
~~Well ord~~ In the set $P(X)$, $\subseteq$ is a linear order.

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Rest part of Q.1.

Well order:→ Let R is a relation in X, then R is called a well-ordering which for each subset A of X has a unique R-first element. Then X is called R-well-ordered.

When R is a partial order then the uniqueness is obtained from antisymmetry.

Let $(X, \leq)$ be any partially ordered set and X has an element a such that for each x in X, $a \leq x$ then a is called the first or least element of X.

Thus a is called the first element of X if and only if a is comparable with all other elements of X and no element of X is a predecessor of a.

Let $(X, \leq)$ be a partially ordered set and $A \subseteq X$. Then $l(\in X)$ is called a lowerbound of A if $l \leq x$ is true for each element x of A.

An element $u(\in X)$ is called an upper bound of $A(\subseteq X)$ if $x \leq u$ is true for each member of x of A.

Let $(X, \leq)$ be any partially ordered set and X has an element a such that for each x in X, $a \leq x$ then a is called the first or least element of X.

Thus a is called the first element of X if and only if a is comparable with all other elements of X and no element of X is a predecessor of a.

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B.Sc-part-II, paper-III, Date-21-04-2020

Differential Calculus (partial differentiation)

Q.1. If $u = \cos^{-1}\frac{x-y}{x+y}$, Prove that $x\frac{\partial u}{\partial x}+y\frac{\partial u}{\partial y}=0$

Soln: We have $\cos u = \frac{x-y}{x+y}$

It is clear that $\frac{x-y}{x+y}$ is of zero degree and hence $\cos u$ is a homogeneous function of ~~degree~~ zero degree.

$\therefore$ By Euler's theorem,

$$x\frac{\partial(\cos u)}{\partial x}+y\frac{(\cos u)}{\partial y}=n\cos u=0\times\cos u=0$$

$$\Rightarrow -x\sin u.\frac{\partial u}{\partial x}-y\sin u.\frac{\partial u}{\partial y}=0 \Rightarrow x\frac{\partial u}{\partial x}+y\frac{\partial u}{\partial y}=0$$ Proved

Q.2. If $u=\sin^{-1}\frac{\sqrt{x}-\sqrt{y}}{\sqrt{x}+\sqrt{y}}$, show that

$$x\frac{\partial u}{\partial x}+y\frac{\partial u}{\partial y}=0$$

Soln. We have $\sin u = \frac{\sqrt{x}-\sqrt{y}}{\sqrt{x}+\sqrt{y}}$

Since, $\sin u$ is a homogeneous function of zero degree.

$\therefore$ By Euler's theorem,

$$x\frac{\partial(\sin u)}{\partial x}+y\frac{\partial}{\partial y}(\sin u)=0\times\sin u$$

$$\Rightarrow \cos u\left(x\frac{\partial u}{\partial x}+y\frac{\partial u}{\partial y}\right)=0$$

$$\Rightarrow x\frac{\partial u}{\partial x}+y\frac{\partial u}{\partial y}=0$$ Proved

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Differential calculus (partial differentiation)

Q.3. If $u = \cos^{-1}\frac{x+y}{\sqrt{x}+\sqrt{y}}$, show that

$$x\frac{\partial u}{\partial x} + y\frac{\partial u}{\partial y} + \frac{1}{2}\cot u = 0$$

Soln: We have, $u = \cos^{-1}\frac{x+y}{\sqrt{x}+\sqrt{y}}$

$$\Rightarrow \cos u = \frac{x+y}{\sqrt{x}+\sqrt{y}}$$

$$\Rightarrow \cos u = \frac{x\left(1+\frac{y}{x}\right)}{\sqrt{x}\left(1+\sqrt{\frac{y}{x}}\right)}$$

$$\Rightarrow \cos u = \frac{x^{\frac{1}{2}}\left(1+\frac{y}{x}\right)}{1+\sqrt{\frac{y}{x}}} = V \text{ (say)} \quad \text{—— I}$$

We find that V is a homogeneous function of two independent variable x and y and of degree $\frac{1}{2}$. Hence applying Euler's theorem for the function V we have

$$x\frac{\partial V}{\partial x} + y\frac{\partial V}{\partial y} = nV = \frac{1}{2}V \quad \text{—— II}$$

From I, $V = \cos u$

$$\therefore \frac{\partial V}{\partial x} = -\sin u\frac{\partial u}{\partial x} \text{ and } \frac{\partial V}{\partial y} = -\sin u\frac{\partial u}{\partial y}$$

Substituting these values in II, we get

$$-x\sin u\frac{\partial u}{\partial x} - y\sin u\frac{\partial u}{\partial y} = \frac{1}{2}\cos u$$

$$\Rightarrow x\frac{\partial u}{\partial x} + y\frac{\partial u}{\partial y} = -\frac{1}{2}\cot u$$

$$\Rightarrow x\frac{\partial u}{\partial x} + y\frac{\partial u}{\partial y} + \frac{1}{2}\cot u = 0.$$

Proved

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Differential calculus (partial differentiation)

Q.4. If $u = \sin^{-1}\left(\frac{x+y}{\sqrt{x}+\sqrt{y}}\right)$, prove that $x\frac{\partial u}{\partial x} + y\frac{\partial u}{\partial y} = \frac{1}{2}\tan u$.

Soln: $\sin u = \frac{x+y}{\sqrt{x}+\sqrt{y}} = v$ (say)

Since v is a homogeneous function of x and y of degree $\frac{1}{2}$

$\therefore$ By Euler's theorem,

$$x\frac{\partial v}{\partial x} + y\frac{\partial u}{\partial y} = nv$$

$$\Rightarrow x\frac{\partial}{\partial x}(\sin u) + y\frac{\partial}{\partial y}(\sin u) = \frac{1}{2}\sin u$$

$$\Rightarrow x\cos u\frac{\partial u}{\partial x} + y\cos u\frac{\partial u}{\partial y} = \frac{1}{2}\sin u$$

$$\Rightarrow x\frac{\partial u}{\partial x} + y\frac{\partial u}{\partial y} = \frac{1}{2}\tan u$$ proved.

Q.5. If $u = \sin(\sqrt{x}+\sqrt{y})$, prove that

$$x\frac{\partial u}{\partial x} + y\frac{\partial u}{\partial y} = \frac{1}{2}(\sqrt{x}+\sqrt{y})\cot(\sqrt{x}+\sqrt{y})$$

Soln: We have, $u = \sin(\sqrt{x}+\sqrt{y}) \Rightarrow \sin^{-1}u = \sqrt{x}+\sqrt{y}$

$$\Rightarrow \sin^{-1}u = \sqrt{x}\left(1+\sqrt{\frac{y}{x}}\right) = V \text{ (say)} \quad \text{--- (I)}$$

We find that V is a homogeneous function of two independent variables x and y of degree $\frac{1}{2}$.

$\therefore$ By Euler's theorem, we get

$$x\frac{\partial v}{\partial x} + y\frac{\partial v}{\partial y} = nv = \frac{1}{2}v \quad \text{--- (II)}$$

From (I), $V = \sin^{-1}u$

$$\therefore \frac{\partial v}{\partial x} = \frac{1}{\sqrt{1-u^2}}\frac{\partial u}{\partial x} \text{ and } \frac{\partial v}{\partial y} = \frac{1}{\sqrt{1-u^2}}\frac{\partial u}{\partial y}$$

Substituting these two values in II, we get

$$\frac{x}{\sqrt{1-u^2}}\frac{\partial u}{\partial x} + \frac{y}{\sqrt{1-u^2}}\frac{\partial u}{\partial y} = \frac{1}{2}\sin^{-1}u$$

$$\Rightarrow \frac{1}{\sqrt{1-u^2}}\left(x\frac{\partial u}{\partial x} + y\frac{\partial u}{\partial y}\right) = \frac{1}{2}(\sqrt{x}+\sqrt{y})$$

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Differential Calculus (partial diff). B.Sc-part-II(H), paper-III 21-04-2020.

Rest part of Q.5.

$$\Rightarrow x\frac{\partial u}{\partial x} + y\frac{\partial u}{\partial y} = \frac{1}{2}\left[\sqrt{x}+\sqrt{y}\right]\sqrt{1-u^2}$$

$$= \frac{1}{2}\left[\sqrt{x}+\sqrt{y}\right]\sqrt{1-\sin^2\left(\sqrt{x}+\sqrt{y}\right)}$$

Hence, $x\frac{\partial u}{\partial x} + y\frac{\partial u}{\partial y} = \frac{1}{2}(\sqrt{x}+\sqrt{y})\cos(\sqrt{x}+\sqrt{y})$ Proved

Q.6. If $u = \sin^{-1}\frac{x^2+y^2}{x+y}$, show that $x\frac{\partial u}{\partial x} + y\frac{\partial u}{\partial y} = \tan u$

Soln: We have, $u = \sin^{-1}\frac{x^2+y^2}{x+y}$

$$\Rightarrow \sin u = \frac{x^2+y^2}{x+y} = \frac{x^2\left(1+\frac{y^2}{x^2}\right)}{x\left(1+\frac{y}{x}\right)} = x\phi\left[\frac{y}{x}\right] = v, \text{ say}$$

Now, v is a homogeneous function of degree 1

$\therefore$ By Euler's theorem, we get

$$x\frac{\partial v}{\partial x} + y\frac{\partial v}{\partial y} = 1.v$$

But $v = \sin u$

$\therefore \frac{\partial v}{\partial x} = \cos u\frac{\partial u}{\partial x}$ and $\frac{\partial v}{\partial y} = \cos u\frac{\partial u}{\partial y}$

Substituting these values in (I), we get

$$\therefore x\cos u\frac{\partial u}{\partial x} + y\cos u\frac{\partial u}{\partial y} = \sin u$$

$$\Rightarrow x\frac{\partial u}{\partial x} + y\frac{\partial u}{\partial y} = \tan u$$ Proved

Q.7. If $\sin u = \frac{x^3+y^3}{x-y}$, show that $x\frac{\partial u}{\partial x} + y\frac{\partial u}{\partial y} = 2\tan u$

Soln: Since, $\sin u = \frac{x^3+y^3}{x-y}$

Here, we see that $\sin u$ is a homogeneous function of the second degree.

$\therefore$ By Euler's theorem, we get

$$x\frac{\partial}{\partial x}(\sin u) + y\frac{\partial}{\partial y}(\sin u) = 2\sin u$$

$$\Rightarrow x\cos u\frac{\partial u}{\partial x} + y\cos u\frac{\partial u}{\partial y} = 2\sin u$$

$$\Rightarrow x\frac{\partial u}{\partial x} + y\frac{\partial u}{\partial y} = 2\tan u$$

Proved

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