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For B. Sc. part (I)

Book: Theory of Equation

Lecture on Descartes rule of signs by

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Descarte's Rule of signs :—

In the equation $f(x)=0$

(i) The number of positive roots cannot exceed the number of changes of signs in $f(x)$.

(ii) The number of negative roots can not exceed the number of changes of sign in $f(-x)$.

positive roots : NO polynomial equation can have more positive roots than there are changes of signs from + to − and from − to +, in its terms of the corresponding expression.

It is difficult to give a rigorous proof of the theorem. However we can verify the statement of the theorem as follows:

To start with, let us take a simple example. The signs of the polynomial

$$x^5 - 5x^4 + 2x^3 + 3x^2 - 5x - 1 = 0$$

are

$\frac{+}{1}$ $\frac{-+}{2}$ $\frac{+--}{3}$

in which there are three changes of signs.

To verify the theorem, let us assume without any loss of generality, that the sign of the polynomial expression are

$+ - + - - + + - - - +$

obviously the given polynomial has six changes of signs

Now, we multiply the polynomial by $x-a$ corresponding to a positive root $a$. The sign of binomial $x-a$ are $+ -$.

The multiplication can be exhibited as follows:

$$\begin{array}{l} + - + - - + + - - - + \\ + - \\ \hline + - + - - + + - - - + \\ \quad - + - + + - - + + + - \\ \hline + - + - \mp - \pm - \mp \mp + - \end{array}$$

There are 4 ambigious signs (viz $\pm$ or $\mp$) in the resulting polynomial and there will be $2^4 = 16$. Combination of these ambiguous signs. The most favourable case is one in which the continuation of sign in the original polynomial remain continuation in the resulting polynomials.