Marwari College Darbhanga

Subject - Physics (Hons)
Class - B.Sc. Part 1
Paper - 01
Group - A
Topic - Bending of Beam
(Properties of Matter)
Lecture Series - 20

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Bending of Beam

Beam → The structure of uniform cross-section, whose length is large as compared to its breadth and thickness. For a such a structure, the shearing stress for any given cross-section is negligible. Beams are used in the construction of bridges and infrastructure where heavy loads are to be supported. They are must commonly used in the structure of multistoried buildings.

Neutral Surface →

When a metallic strip is fixed at one end and loaded at the other a bending is produced due to the moment of the load. The deformation produced by the load brings about restoring forces due to elasticity tending to bring the strip back to its original position.

In equilibrium position

Restoring Couple = Bending Couple

These two couples act in the opposite directions.

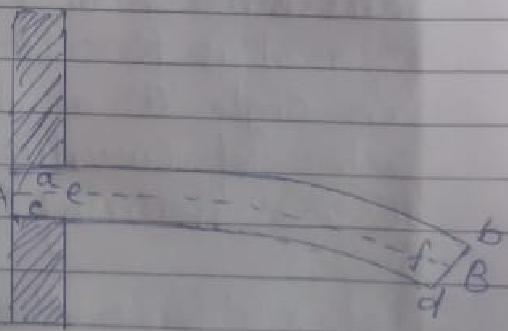

Fig - 1

Suppose a metallic strip consists of a large no. of filaments of small thickness lying one above the other. When a load is applied at the end B, the end A being fixed, inner filament like cd are shortened or compressed while the outer filaments like ab are elongated. Along the section lying in betn these two portions a filament like ef is neither stretched nor compressed. Such a surface is called the neutral surface.

Plane of bending :→ The plane in which bending takes place is known as plane of bending. when the beam is placed horizontally the plane of bending is a vertical plane perpendicular to the beam.

Neutral axis → The Section of the neutral surface (ef) by the plane of bending which is perpendicular to it is called the neutral axis.

* The change in length of any filament is proportional to the distance of the filament from the neutral axis.

Bending Moment.

Consider a small part xy of the neutral axis of the strip bent into an arc of radius R subtending an angle 'θ' at the centre of Curvature O, as shown in fig.-2

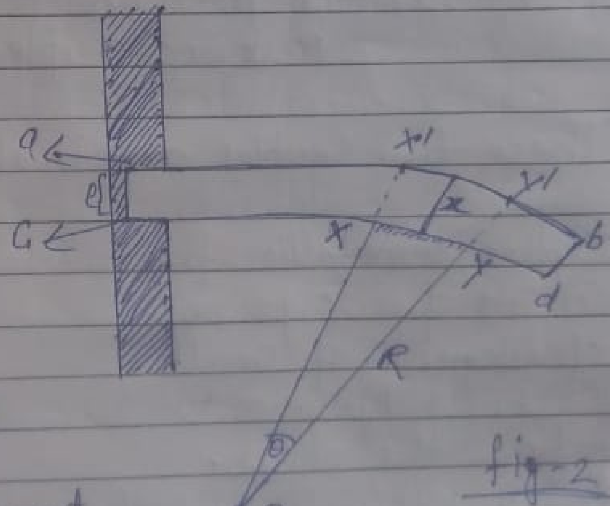

fig-2

Let x'y' be another filament at a distance x from the neutral surface,

then, $xy = R\theta$

and $x'y' = (R+x)\theta$

∴ Increase in length of the filament

$= x'y' - xy = (R+x)\theta - R\theta = x\theta$

$\therefore$ Strain $= \dfrac{\text{change in length}}{\text{original length}} = \dfrac{x\theta}{R\theta} = \dfrac{x}{R}$

Now young's modulus $Y = \dfrac{\text{Stress}}{\text{Strain}}$

$\therefore$ Stress $= Y \times \text{Strain} = \dfrac{Yx}{R}$

Consider a section ABCD of the strip at right angles to its length and the plane of bending (fig. 3)

Then the forces acting on the strip are perpendicular to this Section and the line EF lies on the neutral Surface. The forces producing elongations act in the upper half ABCD and those producing contraction act in the lower CDEF in opposite directions perpendicular to the Section ABCD and hence constitute a couple.

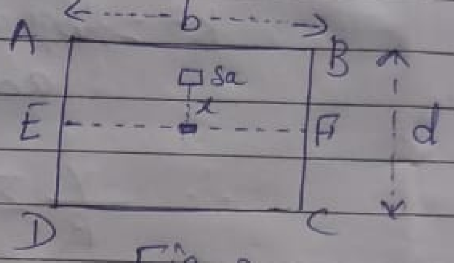

Fig-3

To find the moment of this couple consider a small area δa lying at a distance x from the neutral axis EF,
then

Force on area δa = stress x area $= \dfrac{Yx \cdot \delta a}{R}$

Moment of the force about the axis EF

$$= \frac{Yx\,\delta a\,x}{R} = \frac{Yx^2\,\delta a}{R}$$

Hence, moment of all the forces acting at various points of the whole face ABCD are

$$= \frac{Y}{R} \sum x^2 \delta a$$

To find the value of $\sum x^2 \delta a$,
let us suppose that we can divided the whole area into a no. of such parts each of area δa and let the no. of such parts be n, then

$$\sum x^2 \delta a = x_1^2 \delta a + x_2^2 \delta a + \ldots n \text{ times}$$

$$= n\delta a \frac{x_1^2 + x_2^2 + \ldots + x_n^2}{n} = ak^2$$

where, $a = n\delta a$ = Area of the face ABCD
k^2 = the square of radius of gyration k of ABCD about the axis EF.

$$\therefore \frac{Y}{R} \sum x^2 \delta a = \frac{Yak^2}{R} \quad \text{—— (1)}$$

The quality ak^2 = moment of inertia of the beam if it has a unit mass per unit area and is called the geometrical moment of inertia I.

Hence, moment of the restoring couple $= \dfrac{Yak^2}{R}$

$$= \frac{YI}{R} \quad \text{—— (2)}$$

In equilibrium,
Restoring couple = Bending couple (bending moment)

Hence Bending moment may be defined as the total moment of all the couples arising in a bent beam and trying to resist its deformation caused by an external couple.